Complex Weight Control of Array Pattern Nulling

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ABSTRACT: An efficient method based on the sequential quadratic programming (SQP) algorithm for the linear antenna arrays pattern synthesis with prescribed nulls in the interference direction and minimum side lobe levels by the complex weights of each array element is presented. In general, the pattern synthesis technique that generates a desired pattern is a greatly nonlinear optimization problem. SQP method is a versatile method to solve the general nonlinear constrained optimization problems and is much simpler to implement. It transforms the nonlinear minimization problem to a sequence of quadratic subproblem that is easier to solve, based on a quadratic approximation of the Lagrangian function. Several numerical results of Chebyshev pattern with the imposed single, multiple, and broad nulls sectors are provided and compared with published results to illustrate the performance of the proposed method. © 2007 Wiley Periodicals, Inc. Int J RF and Microwave CAE 17: 304–310, 2007.

Keywords: null steering; antenna array; interference suppression; sequential quadratic programming; pattern synthesis

I. INTRODUCTION

The antenna array pattern synthesis in order to steer nulls to the direction of interference while maintaining the main beam directed to the desired signal has received much attention [1–18]. It plays an important role in communication system, sonar, and radar applications to improve the performance (maximizing signal to interference ratio) and to cancel the jammer signal [12]. Interference suppression in antenna arrays can be achieved by steering nulls in the directions of undesired signals while keeping the main lobe in the direction of user activity by adjusting the excitation amplitude and phase, the excitation amplitude only, the phase only and the position only of the sources constituting the array. Over the past 10 years, several techniques have been developed for antenna array pattern synthesis [7, 11–16] based on a variety of popular methods. Methods of amplitude-only control are easy to implement, amplitude-only control utilize an array of attenuators to adjust the element amplitudes [4, 8, 12]. If the array elements are assumed to be symmetrically even with respect to the center of array, the numbers of variable attenuators required are halved. The phase-only control is of particular interest in null steering, it is responsible for the main beam scanning. The synthesis methods based on the amplitude-only or the position-only control are incapable to realize asymmetric nulls about the main beam [9, 11], the complex coefficient method can impose any asymmetric nulls at arbitrary directions in the side lobe region. This method is effective for the applications in adaptive array with strongly controllable side lobe level (SLL).

Recently, evolutionary algorithms such as particle swarm optimization (PSO) [16], simulated annealing (SA) [6], and genetic algorithms (GA) [7] have been studied for array synthesis including null constrains. The evolutionary algorithms can be considered as a
powerful and interesting technique for solving large kinds of control problems. However, the great disadvantage of the evolutionary algorithms is the great computational cost; require large number of function evaluations to complete the optimization study and the long annealing time.

The sequential quadratic programming (SQP) algorithm has been shown to be an effective alternative optimization algorithm. The SQP algorithm is much easier to implement and suitable for solving nonlinear constrained optimization problems with both equality and inequality constraints, including flexibility to allow nonquadratic cost functions, nonlinear models, and multiple objective optimization. They are considered the most efficient general purpose nonlinear programming algorithms today [19]. The functions and gradients can be evaluated with sufficiently high precision. The basic idea of the SQP optimization is to replace the given nonlinear problem by a sequence of quadratic programming (QP) subproblems that are easier to solve. In this study, [5] the authors presented a highly flexible array synthesis method using the QP method in antenna array pattern synthesis, the SQP methods are iterative methods which solve at each iteration a QP problem which is obtained by linearizing the constraints and approximating the Lagrangian function. Convergence is typically achieved in a few iterations and the optimality criteria method based on Karush-Kuhn-Tucker (KKT) necessary conditions of optimality. The advantage of these techniques is computationally very efficient.

In this study, we examine the performance of SQP algorithm [18–20] for steering the single, multiple, and broad band nulls in the directions of interference by controlling both the amplitude and phase (complex excitation control). SQP algorithm is the generalization of the Newton method for the constrained case. It has an optimum quadratic convergence and gives good optimal solution. The array pattern synthesis problem can be formulated as a nonlinear constrained optimization problem where the optimum complex weight is obtained by solving a QP subproblem at each iteration. The SQP simulated results are validated by comparing with those optimized by minimax approximation in Ref. 9 and Modified Tabu Search Algorithm (MTSA) in Ref. 11.

II. MATHEMATICAL FORMULATION

Consider a linear array composed by 2N equispaced isotropic antenna elements with interelement spacing \( d_0 \). If the element excitations are conjugate-symmetric centric about the center of the array \( w_n = w_2^{*N-n+1} \), the perturbed array pattern can be written as

\[
F(\theta) = \sum_{n=1}^{N} \left( a_n \cos(kd_n \sin \theta) - b_n \sin(kd_n \sin \theta) \right)
\]

Where \( d_n = d_0(2^{N+1} - n) \) and \( w_n = a_n + jb_n \).

Let \( w_n \) is the complex excitation of the \( n \)th element, \( k \) is the wave number, \( d_0 \) is the interelement spacing, \( d_n \) is the \( n \)th element position with respect to the center of the array, \( \theta \) is the angle from broad side, \( a_n \) and \( b_n \) are the real and imaginary parts of the \( n \)th element excitation.

The radiation pattern with prescribed null at the interference direction and minimum SLL is obtained by calculating a set of complex element excitation and approximating the perturbed pattern to the desired pattern i.e. the perturbed pattern should satisfy the following eq. (2), then the template function of the desired pattern can be denoted as

\[
F_d(\theta) = \left\{ \begin{array}{ll}
\delta_i & \theta \in \mathbb{R}_i^i \\
\delta_j & \theta \in \mathbb{R}_j^j \\
F_0(\theta) & \theta \in \mathbb{R}_0
\end{array} \right. \quad i = 1, \ldots, m_x
\]

where, \( \delta_i, \delta_j, F_0(\theta), \mathbb{R}_i, \mathbb{R}_j, \mathbb{R}_0 \) and \( m_x \) are the levels in the regions of the suppressed sectors, the tolerance of the maximum SLL of the perturbed pattern, the initial Chebyshev pattern, the \( i \)th null region, the side lobe region, the angular region of the main beam, and the numbers of nulls prescribed, respectively.

The optimization problem can be modeled by minimization the value of the difference between the perturbed and the desired patterns. Mathematically, the optimization problem can be written as

\[
\min_{\{a_n, b_n\}} |F_d(\theta) - F(\theta)|
\]

\[
f(\theta) = \sum_{n=1}^{N} \left( a_n \cos(kd_n \sin \theta) - b_n \sin(kd_n \sin \theta) \right)
\]
minimize \( f(\theta_0, w) \)
subject to \( f(\theta_i, w) = 0 \) \( i = 1, \ldots, m_e \)
\( f(\theta_j, w) \leq 0 \) \( j = m_e + 1, \ldots, m \)
\( w \in \mathbb{R}^{2N} \) and \( \theta \in \mathbb{R}_0 \)

where
\[
f(\theta, w) = |F_\delta(\theta) - F(\theta)|
= \begin{cases} 
|\delta_i - F(\theta_i)| & \theta_i \in \mathbb{R}^i \ i = 1, \ldots, m_e \\
|\delta_j - F(\theta_j)| & \theta_j \in \mathbb{R}^j \ j = m_e + 1, \ldots, m_e \\
|F_0(\theta) - F(\theta)| & \theta \in \mathbb{R}_0 
\end{cases}
\]
and \( w = [a_1, a_2, a_N, b_1, b_2, \ldots, b_N] \) is the design variable vector, \( f(\theta_0, w) \) is the objective function, \( f(\theta_i, w) \)
is the vector of equality constraints, and \( f(\theta_j, w) \) is the vector of inequality constraints.
\( \theta_0 \) is the angular region of the main lobe, \( \theta_i \) is the \( i \)th directions of interfering signals, \( \theta_j \) is the side lobe region, and \( m \) is the number of the sampled angular direction.

SQP methods are considered as the most efficient methods to solve the problem (3) [18–20]. The Lagrangian of this problem is defined as
\[
L(w, \lambda) = f(\theta_0, w) + \sum_{p=1}^{m} \lambda_p f(\theta_p, w)
\]
where, \( \lambda = (\lambda_1, \ldots, \lambda_m)^T \in \mathbb{R}^m \) is the vector of the Lagrange multiplier.

For the problem (3) at the \( p \)th iteration one solves for the next search direction, the QP subproblem can be defined as
where, $M_p$ is usually a positive semidefinite approximation to the Hessian matrix of the Lagrangian function with respect to $w$. This quadratic subproblem (6) can be solved using any QP algorithm. If $d_p$ is the solution to (6) in iteration $p$, then the solution is used to form a new iterate

$$ w_{p+1} = w_p + z_p d_p $$

(7)

where, $z_p \in [0,1]$ is the step length parameter to enforce global convergence of the SQP method, i.e. the approximation of a point satisfying the necessary KKT optimality conditions. There are different ways that the Hessian matrix of the Lagrangian can be approximated, to avoid calculation of second derivatives and to obtain a final super linear convergence, the Hessian matrix $M_p$ updated by Broyden, Fletcher, Goldfarb, and Shanno (BFGS).

### III. NUMERICAL RESULTS

To demonstrate the validity of the proposed method that synthesizes the array pattern with suppress single, multiple, and broad-band interference signal with the imposed directions and maximum tolerance of SLL using complex current excitations, several computer simulation examples using an equispaced linear array with one half wave interelement spaced 20 isotropic elements were performed. The iteration number was 50. This was sufficient to obtain satisfactory patterns with desired nulling performance. The whole computations took almost 0.25–0.33 s on a personal computer with a Pentium M processor running at 1.6 GHz.

#### Table I. The Complex Current Computed by SQP for Figures 2, 3, 4, and 5

<table>
<thead>
<tr>
<th>Element No.</th>
<th>M</th>
<th>Complex Weights</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.1065 ± j0.0119</td>
<td>0.1132 ± j0.0019</td>
</tr>
<tr>
<td>2</td>
<td>0.1554 ± j0.0106</td>
<td>0.1671 ± j0.0056</td>
</tr>
<tr>
<td>3</td>
<td>0.2736 ± j0.0093</td>
<td>0.2697 ± j0.0030</td>
</tr>
<tr>
<td>4</td>
<td>0.3898 ± j0.0082</td>
<td>0.3780 ± j0.0030</td>
</tr>
<tr>
<td>5</td>
<td>0.5054 ± j0.0067</td>
<td>0.5130 ± j0.0056</td>
</tr>
<tr>
<td>6</td>
<td>0.6409 ± j0.0058</td>
<td>0.6534 ± j0.0019</td>
</tr>
<tr>
<td>7</td>
<td>0.7775 ± j0.0042</td>
<td>0.7710 ± j0.0040</td>
</tr>
<tr>
<td>8</td>
<td>0.8836 ± j0.0033</td>
<td>0.8811 ± j0.0053</td>
</tr>
<tr>
<td>9</td>
<td>0.9570 ± j0.0017</td>
<td>0.9672 ± j0.0006</td>
</tr>
<tr>
<td>10</td>
<td>1.0000 ± j0.0008</td>
<td>1.0000 ± j0.0008</td>
</tr>
</tbody>
</table>

Figure 6. Pattern synthesis with a wide sectors imposed around −60°.

Figure 7. Pattern synthesis with four nulls imposed at −45, −28, 14, and 35°.
An initial 40-dB Chebyshev is assumed for a 20 equispaced linear elements with \( \lambda/2 \) interelement spacing, as shown in Figure 1.

Figure 2 shows the radiation pattern obtained by controlling both the amplitude and phase with one imposed null at \( 20^\circ \) and the constrained null depth. The maximum SLL and the null depth are \(-37.5\) and \(-200\) dB, respectively. In the second example, we have shown the perturbed pattern with double nulls imposed at 14 and \( 40^\circ \), and the nulls depths are 90 and 77.5 dB, respectively. The SLL of the Figure 3 is 40 dB. In the Figure 4, triple nulls forced at \( 34, 21 \), and \( 40^\circ \). The nulls are deep around 69, 69.2, and 64.5 dB level below the maximum, respectively.

The radiation pattern plot in Figures 5–7 obtained by using SQP demonstrates the capability of this technique to prescribe a wide band interference signal. Figure 5 shows the nulling patterns with two prescribed wide sectors when their centers are imposed at \(-20\) and \(-42^\circ \). The desired nulls are deeper than 62 dB. The computed element complex current excitations for Figures 2, 3, 4, and 5 are given in Table 1. The pattern with a broad null sector imposed when their center at \( -60^\circ \) shows in Figure 6. The corresponding bandwidth and null depth are \( 22\% \) and 73 dB, respectively. In Figure 7, we have shown the nulling pattern with four imposed nulls depth at the direction of the interfering signal. The corresponding angles are \(-45, -28, 14, \) and \( 35^\circ \), respectively. Table II gives the computed element complex current for the Figures 6, 7, 8, and 9.

In Figures 8 and 9, we have shown the numerical results calculated by SQP algorithm in comparison with other methods, such as the minimax approximation [9] and MTSA [11]. For comparison, an initial 30-dB Chebyshev is assumed for 20 equispaced linear elements with \( \lambda/2 \) interelement spacing is considered.

### Table II. The Complex Current Computed by SQP for Figures 6, 7, 8, and 9

<table>
<thead>
<tr>
<th>Element No. M</th>
<th>Complex Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Figure 6</td>
</tr>
<tr>
<td>±1</td>
<td>0.0928 ± j0.0034</td>
</tr>
<tr>
<td>±2</td>
<td>0.1819 ± j0.0121</td>
</tr>
<tr>
<td>±3</td>
<td>0.2573 ± j0.0138</td>
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<tr>
<td>±4</td>
<td>0.3800 ± j0.0101</td>
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<tr>
<td>±5</td>
<td>0.5144 ± j0.0044</td>
</tr>
<tr>
<td>±6</td>
<td>0.6433 ± j0.0001</td>
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<tr>
<td>±7</td>
<td>0.7690 ± j0.0009</td>
</tr>
<tr>
<td>±8</td>
<td>0.8834 ± j0.0013</td>
</tr>
<tr>
<td>±9</td>
<td>0.9520 ± j0.0049</td>
</tr>
<tr>
<td>±10</td>
<td>1.0000 ± j0.0075</td>
</tr>
</tbody>
</table>

**Figure 8.** SQP optimized radiation pattern (solid line) obtained by controlling both the amplitude and phase with a broad null (\( \Delta \theta = 5^\circ \)) centered at 30° compared with the result by MTSA [11] (dotted line).

**Figure 9.** SQP optimized radiation pattern (solid line) obtained by controlling both the amplitude and phase with three wide sectors imposed at the \( \sin(\theta) = -0.35, 0.35, \) and 0.75 compared with the result by Minimax approximation [9] (dotted line).
In Figure 8, the array pattern obtained by controlling both the amplitude and phase with a broad null sector centered at 30° with relative bandwidths of 5%. This example was calculated using MTSA [11] (see Fig. 9 in Ref. 11). The array excitations optimized by MTSA and SQP algorithm have complex-conjugate symmetry about the center of the array. The excitations obtained by MTSA are listed in Table II in Ref. 11. The desired null depth was chosen according to the MTSA simulated results and which are 70 dB. Figure 9 shows the comparison of simulated radiation pattern obtained by minimax approximation in Ref. 9 (see Fig. 1 in Ref. 9) and our SQP simulation. In Figure 9, it can be seen that the simulation result agree well on the null positions (centers at −0.35, 0.35, and 0.75), relative bandwidths (5, 5, and 10%, respectively), and null depth. However, the maximum SLL of the perturbed pattern is not maintained at 30 dB (the corresponding SLL is −28.5 dB). The excitations obtained by minimax approximation are listed in Table I in Ref. 9. The computed element excitations optimized by SQP algorithm for Figures 6, 7, 8, and 9 are given in Table II. The excitation given in Tables I and II are normalized with respect to the excitation of the center elements.

IV. CONCLUSION

A method for antenna array pattern synthesis based on the SQP algorithm has been presented to form nulls at the directions of interfering signals by controlling the amplitude and the phase of each array element. The numerical results show that the complex excitation control using SQP algorithm is efficient for prescribed single, multiple and broad nulls, and control the null depth and maximum of sidelobe level. The SQP algorithm can be applied to nonisotropic elements array with different geometries including amplitude-only and phase-only control.

REFERENCES

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